How to Study for a Mathematics Exam

By Emily T. Winn

In high school, one never really "studied" for a mathematics exam, save maybe look over a few hard problems from the homework or work on a study guide the teacher handed to you. Most of us got by just on completing homework and paying attention in class; the tests were just more regurgitation. In college, however, after the first courses of calculus, correctly preparing for an exam in mathematics is crucial to not only excelling on an exam but also learning the math in the first place. Most of us were not taught how to do this in high school, so here I present my tips for navigating exams in college and beyond.

How long does this take? I recommend starting to study at least a week ahead of the exam because it allows you to organize your materials and clarify your weak points. In practice, for many this is not possible. The best thing you can do for yourself is schedule/block out time to study and commit to what you can do.

1. Memorize all definitions cold. You should know definitions so well that if someone shook you awake at 4 am and asked you to state a definition, you would recite it perfectly without even considering how rude it was to be woken up at that hour. Flashcards are your friends here. (Or Quizlet, if you prefer online/mobile studying.)

   But what good does this do?
   a. You won't have to waste precious time on an exam trying to remember a definition or figure out what a question is asking. If the test is open notes/books, you won't waste time flipping through pages trying to find what that one word means.
   b. Some professors will directly ask for definition statements. Easy points and confidence boost.
   c. Most professors will offer partial credit for an extensive word problem or proof if you include a correct definition. Pick up a couple points even if you are blanking on what to do.
   d. If you have a notecard or paper for a test, you don't have to waste space on definitions and can save it for information that is harder to memorize, like complicated formulas.

2. Memorize all theorem statements cold. Start with the statements that have specific names (e.g., Weak Law of Large Numbers, Monotone Convergence Theorem, etc.). Memorize these cold like you do the definitions. Then, memorize the rest that were stated in class if you can (including propositions, corollaries, and lemmas). Prioritize the ones the professor emphasized in class and the ones you saw most often on the homework.

   And how does this help?

a. Some professors ask for theorem statements, fill in the blank questions, or true/false questions pertaining to the unnamed statements. Again, you can quickly pick up points and get a confidence boost.
b. You will most likely be asked to prove something on an exam. Every proof starts with given assumptions. Use those assumptions to prompt your memory of any relevant theorems that could connect to the correct proof; use those to brainstorm and put together your answer. (See below for more tips on how to approach a proof you've never seen before.)
c. Many professors offer partial credit if you write down relevant theorems. You can pick up points if your intuition is correct but you're running out of time. (Ex. "This somehow relates to the Weak Law of Large Numbers" will get you a point or two if the correct answer for the problem includes WLLN.)

3. <u>DO NOT MEMORIZE PROOFS.</u> Did your professor tell you that the proofs from class are fair game, or give you a short list of proofs that are fair game for the test? Do not memorize them word for word. You will do too much work for too little gain by trying to do that and you won't actually understand the math behind the proofs. Rather, memorize the barebones outline for the proofs and draw from that.

   Why is this effective?
   a. Memorizing a handful of bullet points is way easier than memorizing pages of text.
   b. As you learn the barebones outlines, you start to see patterns for proof strategies, such as induction, diagonalization argument, proof by contradiction, or when to use certain theorems. Now, you are learning the actual techniques, which will train you for when you confront a proof on the exam you've never seen before.
   c. Say it with me: partial credit. If you are on an exam and are asked to take a proof from class, you can jot the bullet points if you are on a time crunch. Even if you have time to write the whole proof, jotting the bullet points will help cue you for what your next steps in the proof should be. Finally, if you forget one of your points, you can flesh out the rest of them and note that you are missing a step – in doing so, you'll gather most of the points on the problem.

4. <u>Practice, practice, practice!</u> The best way to learn math is to do it. Focus on concepts that you did not quite understand the first time around – it's very easy to only study what you know so you feel good, but this only gives you a false sense of knowledge going into the exam! There are many ways to do this:
   a. Redo any homework, exam, or quiz problems you got wrong. Compare to the answers.
   b. Do extra textbook problems or any extra/bonus problems the professor gave you. (Pro tip: Professors often use their textbooks or extra problems for inspiration in creating their exams!)
   c. Teach problems or proofs to other classmates or friends (even if the friends aren't in math!). If you don't have friends, talk to yourself out loud. Not

having friends should never stop you from success.

5. Get plenty of sleep the couple nights before the exam. Yes, this is cliché. But you know what happens when you're not well rested? You drop negative signs. You make arithmetic errors. You lose just enough points to drop half a grade. Don't do that to yourself – set yourself up to get the best grade possible and give your big brain a proper break before exam time.

But wait! What if I have an index card or a sheet of paper? What should I put on that? Does this mean I don't have to study?

Having an extra guide on the test is always welcome! But there are catches, like a false sense of security, relying too much on the card/paper, or assuming you don't have to study. Let's break down your questions:

1. What should I put on my index card? A lot of this will depend on the class you are in, and how you work as a learner. The following was my strategy but it may vary if different things come easier to you.
    a. Start with complicated formulas (for example, probability distributions) and long theorems that are hard to memorize. If applicable, include bullet points for the theorem proofs (way less space than the full statements!).
    b. Next, anything on the exam you might overthink. For example, in my probability exams, I always put Fatou's Lemma and Jensen's Inequality on my cards because I always overthink where the inequality sign points.
    c. Finally, any proof strategies that came up most in the homework assignments.
    d. If you still have room, more factoids/statements from the notes or homework that could come in handy for proofs.
    e. Bonus tip: Take the time to make your card neat! A little organization can go a long way.
        i. Write in blue or black ink so that the card doesn't fade. If you have large or messy handwriting, consider typing up your card for easy reading.
        ii. Use a straight edge to box/group sections of your card. Use a highlighter to help you find sections easily. (For example, a whole box for the gamma distribution formulas, labeled "Gamma Dist.")

2. Does this mean I don't have to study? Trick question – you study while you make an index card. That said, you should still study as in steps 1-5 above. Additionally:
    a. Study first, then make your card. When you study, you learn what the major concepts are and what will be emphasized on the exam. This will help you prioritize what to put on the card. Additionally, you'll learn what you know very well and what you still haven't grasped. Again, this will help you prioritize what to put on your card/paper.
    b. Try practice problems with your card. Select and try few problems you've never seen before with what you have on your card (ideally, do this while you still have room to add things). Does it suffice? Do you have enough? This

will prevent the sinking feeling of "I knew I should have put that on there" during the exam.

c. What if it's open notes? No card = no studying! False. You still have to study for the exam as in steps 1-5. Further, you should organize your notes
   i. Highlight key theorems, definitions in your notes/textbooks. Again, this makes them easy to find during the exam.
   ii. Use tabs or post it notes to mark sections. That way you're not wasting time flipping through. Is there a problem on F-distributions? Go to the F-distributions tab, all you know is there. Done.
   iii. Organize homeworks too. Put them in order, label at the top what concepts were covered, and highlight where problems start. Homework assignments are great sources of inspiration when you think you've run out of ideas.

HOW TO DO THAT PROOF YOU'VE NEVER SEEN BEFORE

(P.S.: this scenario is HIGHLY likely if you have an index card or are allowed to refer to your notes.)

You've prepared for the exam. You've studied your butt off and reviewed everything. You're cranking out the first couple problems. Things are great! You got this! And then you turn the page to see… something you've never seen before. It's a proof that was not in class and not in homework assignments. One that requires you to *think* on the spot. How do you do this without Google, your classmates, office hours?

1. Relax. Professors use exam problems no one in the class has seen before on purpose – it's their way of seeing who truly understands the math and who just memorized certain problems. It's also a metric of what portion of the class understood a certain concept. They probably anticipate this is difficult and that not everyone will get it. So do what you can in the allotted time.
2. Think: What is this problem asking? What are the relevant assumptions? What are the definitions of the fancier/new words from class? Did they say to use a certain theorem or technique? (Ex. "Use the Weak Law of Large Numbers to show ____.") What does that theorem/technique mean or say? This part is half the battle!
3. Use your intuition – what are the relevant theorems/ideas? Does this remind you of a homework problem? A particular theorem relating the assumptions and the conclusion? Can you use theorems to link the assumptions to a different set of assumptions of a theorem that gives your conclusion? Are the definitions related? Use these to brainstorm. Think of these as your "puzzle pieces" some of which will link together to put together the proof. (BONUS: if you write down some of your instincts and it's correct, you still may pick up a few points!)
4. Try something! Pick whichever "lead" you think is the strongest and go in that direction. Sometimes things fall out as you actually do the proof. Use scrap paper to your advantage here.
5. If you are really stuck, skip and come back. Sometimes the best way to get something on an exam is to do an initial brainstorm, go do something else (either another problem on the exam, if timed, or take a break if it's take home), and then come back to it with fresh eyes. Again, if time is running out and you don't have

anything, write down what your general intuition is to demonstrate you are thinking and have ideas – this will get some partial credit and it shows the professor you actually had ideas for approaching the problem.

Great! So I just do this and then I get an A?

Hopefully, yes. However, sometimes even if we study our hardest, we don't perform as well as we hope to. There are lots of reasons for this. Maybe your life circumstances are affecting your ability to think clearly or are preventing you from spending the time necessary to learn the material. Maybe you're in a class with students who have taken more math classes, and therefore have had more practice taking these exams. Maybe a disability prevents you from performing well in a given time constraint but for whatever reason, you were unable to secure extra time. Maybe this is the first time you've seen this kind of mathematics with proofs and theorems, and you've reached a level where it takes people two or three chances to learn the material before it sticks. Every mathematician at some point has performed short of what they wanted. (And if you meet someone who has never had this issue, this person is either lying or is a future Fields Medalist.)

The important thing about these study techniques is that they help you learn not just for the exam but also hold onto that information as you move on to more math classes. You want to be able to recall these definitions and theorems from the course you're in now that will count as a prerequisite for something else. Even if you don't have that kind of memory, you'll have materials organized to go back and refresh yourself on the information in the future. Math, like so many fields, takes grit and tedious work to master; and like so many fields, whether someone succeeds comes down to the time and effort they are willing and able to put into the subject, not their innate abilities. If you don't do well, use it to learn and become a better mathematician.